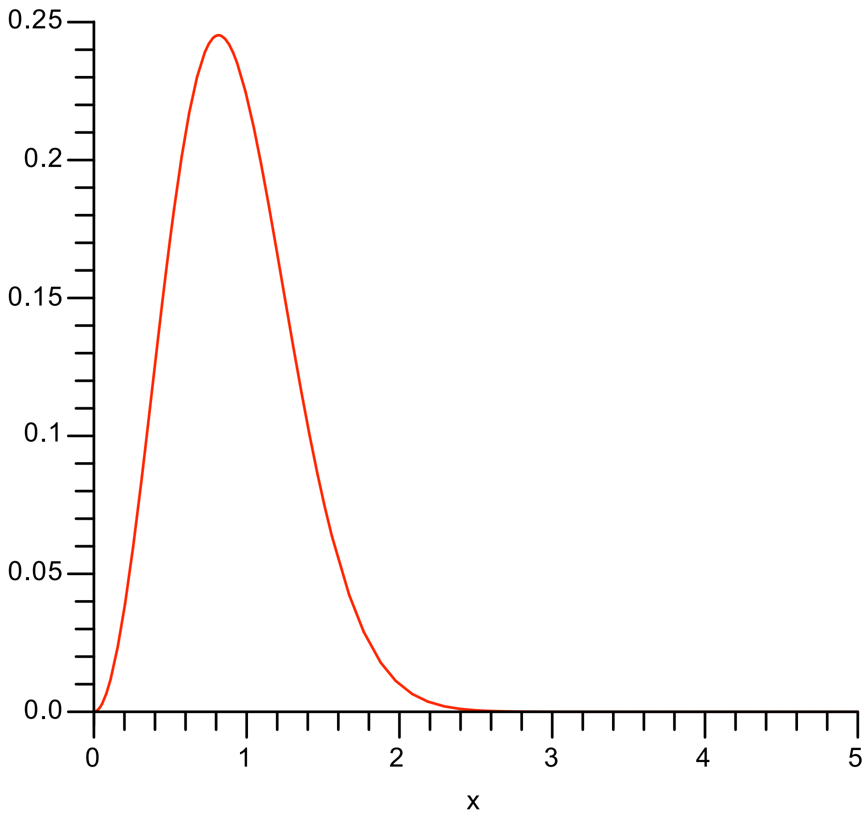
Homework #1: In addition to solving the two requird integrals, I will expand this to show how you can use Maple to mathematically "probe".

First let me plot the integrand:

$$plot\left(x^2 \cdot exp\left(\frac{-3 \cdot x^2}{2}\right), x = 0 ..5\right)$$



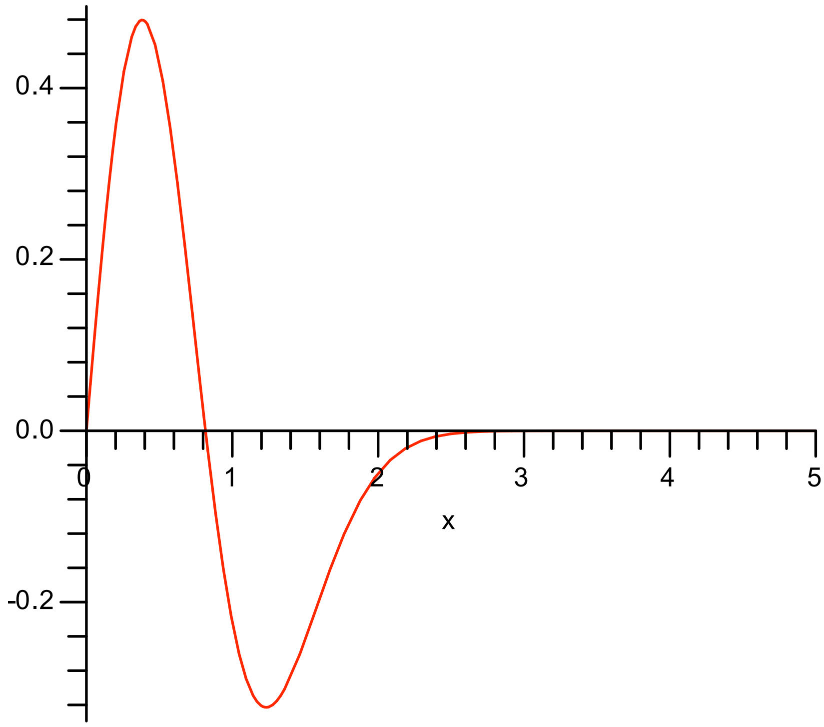
Here you see that the function that you are to integrate is maximum at roughly 1 and quickly fall to near zero before x=3. I can find that maximum very easily by
defining a function f(x) which is the derivative of the integrand with respect to x.

$$f := x \rightarrow diff\left(x^2 \cdot exp\left(\frac{-3 \cdot x^2}{2}\right), x\right)$$

$$x \rightarrow \frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 \, \mathrm{e}^{\left(-\frac{3}{2} x^2\right)}\right) \tag{1}$$

Now I can plot the deriative of the integrand and show that where f(x)=0, there is an extremum (either a minimum or maximum).

$$plot(f(x), x = 0 ..5)$$



I can solve for the value of x at which the f(x)=0, or the integrand is maximum

$fsolve(f(x) = 0);$

$$0. \tag{2}$$

But this returns the trivial result, so I need to specify that I am loking for the solution between 1/2 and 2

$fsolve(f(x) = 0, x = 0.5 .. 2);$

$$0.8164965809 \tag{3}$$

Now let me do the first integral, which corresponds to the total area under the curve of in figure 1:

$int\left(x^2 \cdot exp\left(\frac{-3 \cdot x^2}{2}\right), x = 0 .. \infty\right);$

$$\frac{1}{18}\sqrt{3}\sqrt{2}\sqrt{\pi} \tag{4}$$

Now if I want a numerical result

$evalf(\%);$

$$0.2412004182 \tag{5}$$

Now let me do the second integral, which corresponds to the area under the curve up to x=1:

$int\left(x^2 \cdot exp\left(\frac{-3 \cdot x^2}{2}\right), x = 0 .. 1\right);$

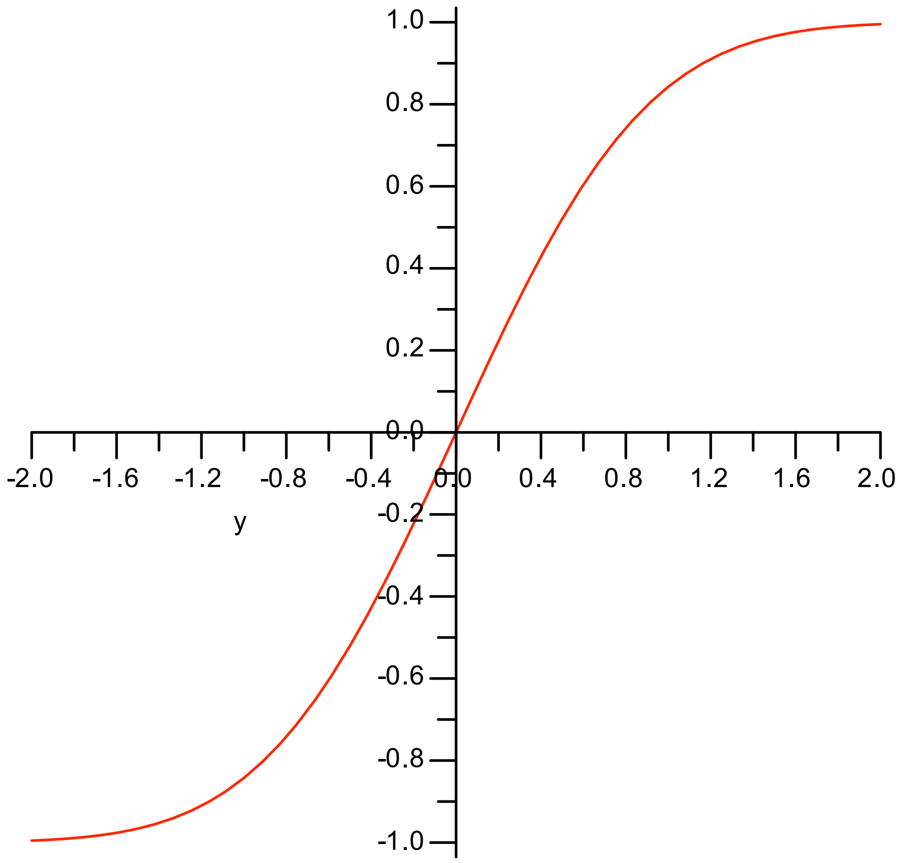
$$-\frac{1}{3}\,e^{\left(\frac{-3}{2}\right)}+\frac{1}{18}\sqrt{\pi}\,\sqrt{6}\,\mathrm{erf}\left(\frac{1}{2}\sqrt{6}\right) \tag{6}$$

$evalf(\%\ );$

$$0.1467402620 \tag{7}$$

Erf refers to the error function and it looks like this:

$plot(erf(y),\ y=-2\,..2\,);$



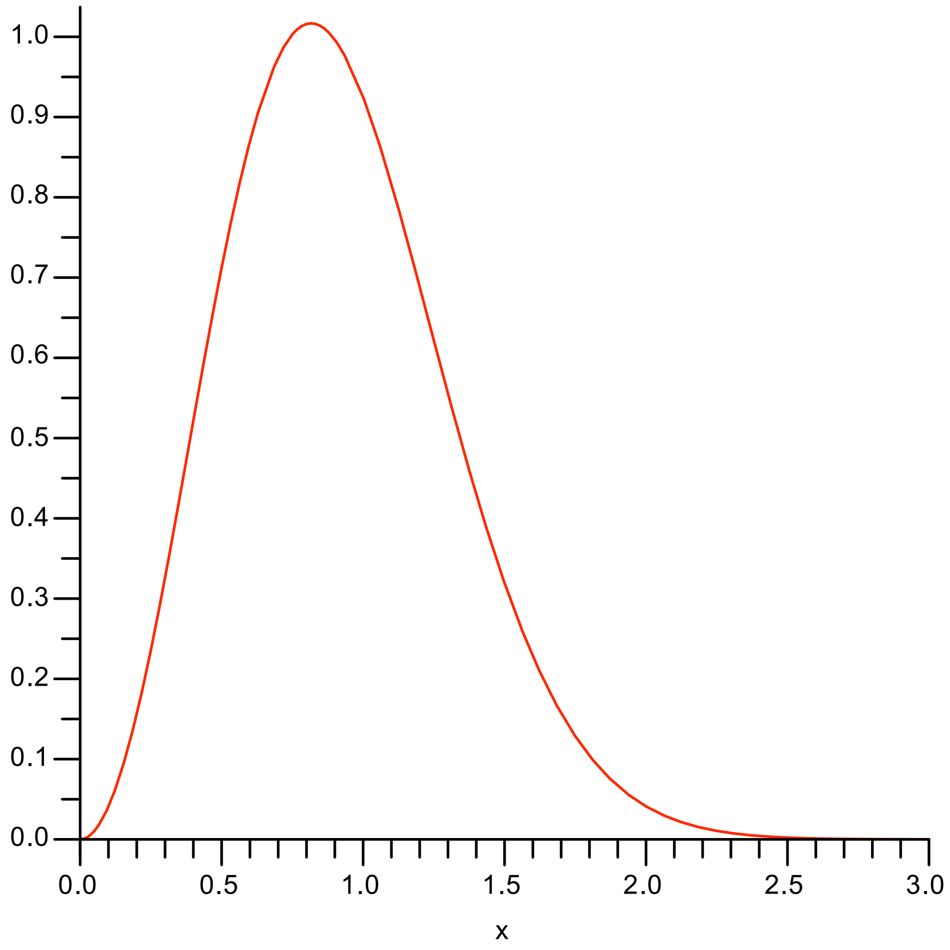
Now it ends up that the integrand is an unnormalised probability distribution that we will be dealing with. Let us call P(x) the
normalised distribution.

$$P := x \rightarrow\ x^2\cdot\ \frac{exp\left(\frac{-3}{2}\cdot x^2\right)}{int\left(x^2\cdot\ exp\left(\frac{-3}{2}\cdot x^2\right),\ x=0\,..infinity\right)};$$

(8)

$$x \rightarrow \frac{x^2\, e^{\left(-\frac{3}{2}x^2\right)}}{\int_0^{\infty} x^2\, e^{\left(-\frac{3}{2}x^2\right)}\, dx} \tag{8}$$

plot$(P(x), x = 0..3)$;



evalf$(P(x))$;

$$4.145929794\, x^2\, e^{(-1.500000000\, x^2)} \tag{9}$$