

Statistical Mechanics  
of  
Nonequilibrium Liquids

by

Denis J Evans and Gary P Morriss

Research School of Chemistry,  
Australian National University,  
Canberra, ACT  
Australia

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## List of Symbols

### Transport Coefficients

$\eta$	shear viscosity
$\lambda$	thermal conductivity
$\eta_V$	bulk viscosity
$\zeta$	Brownian friction coefficient
$D$	self diffusion coefficient

### Thermodynamic Fluxes

$\mathbf{P}$	the pressure tensor
$\mathbf{J}_Q$	heat flux
$\mathbf{\Pi}$	viscous pressure tensor

### Thermodynamic forces

$\nabla \mathbf{u}$	strain rate tensor
$\gamma$	shear rate, $=\partial u_x / \partial y$
$\nabla T$	temperature gradient
$\nabla \cdot \mathbf{u}$	dilation rate
$\mathbf{u}$	streaming velocity
$\boldsymbol{\epsilon}$	elastic deformation
$\nabla \boldsymbol{\epsilon}$	strain tensor
$\text{dot}(\boldsymbol{\epsilon})$	dilation rate, $=1/3(\nabla \cdot \boldsymbol{\epsilon})$

### Thermodynamic State Variables

$T$	temperature
$k_B$	Boltzmann's Constant
$\beta$	$1/k_B T$
$V$	volume
$p$	hydrostatic pressure, $=1/3 \text{tr}(\mathbf{P})$
$N$	number of particles
$\rho$	mass density
$n$	number density

### Thermodynamic Constants

$G$	shear modulus
$C_V$	constant volume specific heat
$C_p$	constant pressure, specific heat
$c_V$	constant volume, specific heat per unit mass
$c_p$	constant pressure, specific heat per unit mass
$D_T$	isochoric thermal diffusivity

## Thermodynamic Potentials

$E$	internal energy
$U(\mathbf{r},t)$	internal energy per unit mass
$S$	entropy
$s(\mathbf{r},t)$	internal energy per unit volume
$\sigma$	entropy source strength = rate of spontaneous entropy production per unit volume
$I$	enthalpy
$Q$	heat

## Mechanics

$L$	Lagrangian, seldom used
$H$	Hamiltonian
$H_0$	phase variable whose average is the internal energy
$I_0$	phase variable whose average is the enthalpy
$J(\Gamma)$	dissipative flux
$F_e$	external field
$\alpha$	thermostatting multiplier
$iL$	p-Liouvillean
$iL$	f-Liouvillean
$A^\dagger$	Hermitian adjoint of $A$
$\Lambda$	phase space compression factor
$\exp_R$	right time-ordered exponential
$\exp_L$	left time-ordered exponential
$U_R(t_1,t_2)$	incremental p-propagator $t_1$ to $t_2$
$U_R(t_1,t_2)^{-1}$	inverse of $U_R(t_1,t_2)$ ., take phase variables from $t_2$ to $t_1$ $U_R(t_2,t_1) = U_L(t_1,t_2)$
$U_R(t_1,t_2)^\dagger$	incremental f-propagator $t_1$ to $t_2$
$T_R$	right time-ordering operator
$T_L$	left time-ordering operator
$C_{AB}(t)$	equilibrium time correlation function, = $\langle A(t)B^* \rangle$
$[A,B]$	commutator bracket
$\{A,B\}$	Poisson bracket
$\delta(t)$	Dirac delta function (of time)
$\delta(\mathbf{r}-\mathbf{r}_i)$	Kirkwood delta function $= 0, \quad  \mathbf{r}-\mathbf{r}_i  > \text{an 'infinitesimal' macroscopic distance, } l$ $= 1/l^3, \quad  \mathbf{r}-\mathbf{r}_i  < \text{an 'infinitesimal' macroscopic distance, } l$
$f(\mathbf{k}) = \int d\mathbf{r} e^{i\mathbf{k}\cdot\mathbf{r}} f(\mathbf{r})$	spatial Fourier transform

$f(\omega) = \int_0^\infty dt e^{-i\omega t} f(t)$	temporal Fourier-Laplace transform
$d\mathbf{S}$	infinitesimal vector area element
$\mathbf{J}^\perp$	transverse momentum current
$\Phi$	total intermolecular potential energy
$\phi_{ij}$	potential energy of particle $i,j$
$K$	total kinetic energy
$f_c$	canonical distribution function
$f_T$	isokinetic distribution function
$m$	particle mass
$\mathbf{r}_i$	position of particle $i$
$\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$	
$\mathbf{v}_i$	velocity of particle $i$